Title: Global Cauchy theorems

Abstract: Our study begins with the following well known consequence of the 'Cauchy-Goursat theorem': for a given open $U \subseteq \mathbb{C}$ and a closed path γ (assumed to be of class C^1) in U, one has

$$\int_{\gamma} f = 0 \text{ for every analytic function } f \text{ on } U. \tag{1}$$

The aim of this series of lectures is to seek global and general conditions on how U and γ are to be related to each other, instead of imposing some stringent condition on U or γ alone, so that (1) holds true. Thus we are led to what is usually referred to as the 'Global versions of Cauchy's Theorem'.

We first prove the *Homotopic* version of Cauchy's theorem that provides a topological relation between U and γ. After that, if time permits, we take up this theme in the utmost generality, namely the *Homology version* of the same. Indeed, the later can be realized as the most general form of the well known 'Cauchy's integral formula'. So from a different perspective, the aim of this discussion can be put as to view the well known formula in its full generality.

Brief plan: We begin the lecture series with the notion of *Homotopy of curves.* Then we prove the homotopic version of Cauchy's theorem. Now we proceed towards the homology version. For this, we plan to recall the basic properties of complex exponential and logarithm. Next we introduce the notion of *winding number of a closed curve w.r.t. a point.* After that we talk about *homology of paths* and see that homotopic paths are homologous. Then we state the Homology version of Cauchy's theorem and devote the rest to Dixion's proof [1]. Note that the extent of the later depends upon the availability of time.

References

[1] John D. Dixon, *A brief proof of Cauchy's integral theorem*, Proc. Amer. Math. Soc **29** (1971), 625-626